

# Consensus for Formation Control of Nonholonomic Mobile Robots

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**Abstract**—In this article we present novel formation control laws based on artificial potential fields and consensus algorithms for a group of unicycles enabling arbitrary formation patterns for these nonholonomic vehicles. Given connected and balanced graphs we are able to prove stability of the rendezvous controller by applying the LaSalle-Krasovskii invariance principle. Further, we introduce obstacle avoidance, enabling a reactive behavior of the robotic group in unknown environments. The effectiveness of the proposed controllers is shown using computer simulations and finally, a classification w.r.t. existing solutions is done.

## I. INTRODUCTION

Lately, the field of cooperative control for robotic systems is of increasing interest to the scientific community. Driven by potential applications, such as environmental monitoring and surveillance or the control of satellite formations, the goal is to develop a distributed control strategy for a group of agents such that the aggregate system attains a prespecified task including consensus, coverage, flocking or formation control [14], [16].

Common to all these problems is that the control laws, due to their distributed nature, feature a crucial dependency on the information flow between the vehicles. This information flow is represented as a graph and the properties of this graph influence the stability of the aggregate system.

Other than in the case of flocking or swarming, where only a loose coupling between the individual agents is needed, the problem of formation control of a group of agents is to assign a geometric structure or pattern to the group, such that the agents attain this formation and keep it over time. Therefore, a strong, prespecified coupling between the agents is needed to accomplish this goal. So the task could be divided into two parts: i) given an initial distribution of agents, set up the desired formation and ii) keep this formation while traveling through the environment.

Conceptually, generating the formation could be done using consensus algorithms [16], [17]. These are used to acquire an agreement within the group of agents. To this end, a control law, based on the information exchange within the group, drives each agent to a position fulfilling the agreement. The formation itself is defined using interaction

forces between the vehicles generated by artificial potential fields [5], [11].

Further, in most of the previous work the agents were modeled as simple double integrators, which seems appropriate for fully actuated, unconstrained robotic systems. In contrast, mobile robots (unicycles, cars, etc.) do not belong to this class of systems as they typically have nonholonomic constraints restricting their instantaneous movement. Thus, many of the existing results are generally not applicable to these systems, but in recent work, progress in the coordination of such vehicles was made [3], [4], [7],[12], [13].

Hence, our goal is to solve the formation control problem for a group of unicycles that are connected by a communication network. We apply a rendezvous and formation control law to such a group by using consensus based algorithms and artificial potential fields. After introducing the necessary preliminaries in Section II we examine the case of rendezvous in Section III, where the initially distributed agents will agree to travel towards a common point and stay there. We then extend this controller to the case of creating fixed formations for the agents in Section IV, where the interaction forces between the agents are derived as the gradients of special artificial potential functions, introduced in [5]. The resulting controllers stabilize the formations and hence, the movement of the whole group. Finally, we integrate obstacle avoidance to the system in Section V, such that each agent is, based on a switching logic, able to avoid obstacles that are within its sensing range. In the event of detection the formation control is switched off and the avoidance scheme secures the vehicle until it has passed the obstacle. This results in a novel type controller, feasible to stabilize nonholonomic vehicles and enabling obstacle avoidance for every agent. Therefore, this reactive system is useful in robotic applications such as navigation in unknown environments. Before we conclude the paper in Section VII, a comparison, classifying the results w.r.t. the related work, is done in Section VI.

## II. PRELIMINARIES

We will introduce some basic notions used throughout the paper concerning the dynamics of the agents under examination, relevant graph theory and the potential functions used for the control laws.

### A. The Unicycle model

The robots under study are *unicycles*, kinematic systems having two driven wheels (back) and one castor wheel (front) as depicted in Fig. 1. Due to the nonholonomic motion constraint it is impossible to stabilize these systems using smooth time-invariant feedback laws [1].

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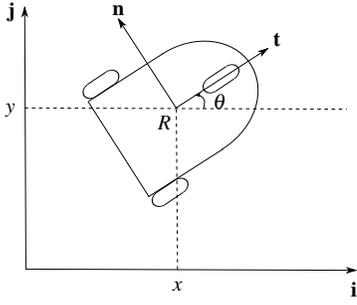


Fig. 1. Unicycle model

Given a fixed orthonormal inertial basis  $[\mathbf{i}, \mathbf{j}]$  in the plane of motion, we pick a body fixed coordinate chart  $[\mathbf{t}, \mathbf{n}]$  attached to the unicycle in  $R$ . Then the position vector of this point can be described as  $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$  and the orientation of the body fixed frame w.r.t. the inertial frame is given by the angle  $\theta$ . Due to the differential drive of the back wheels, its dynamics are

$$\begin{aligned} \dot{x}_i &= u_i \cos \theta_i, \\ \dot{y}_i &= u_i \sin \theta_i, \\ \dot{\theta}_i &= \omega_i, \end{aligned} \quad (1)$$

where  $\omega_i$  and  $u_i$  are considered as the inputs of the system and denote the angular velocity and the velocity of the unicycle, respectively.

### B. Graph theory and network topology

As information exchange is fundamental to cooperative control, graphs are used to represent the network topology of a group of agents. We provide a condensed definition and refer to [6] for a thorough treatment of graphs and their properties.

*Definition 1:* A graph  $\mathcal{G}$  is a finite set of elements  $\mathcal{V}(\mathcal{G}) = \{v_1, \dots, v_n\}$ , the *vertices* of a graph, and a set  $\mathcal{E}(\mathcal{G}) \subset \mathcal{V} \times \mathcal{V}$  called the *edges* of a graph. If for all  $(v_i, v_j) \in \mathcal{E}$ ,  $(v_j, v_i) \in \mathcal{E}$  as well, the graph is said to be *undirected*, otherwise it is called *directed*. The *in-degree* of a vertex  $v \in \mathcal{V}$  defines the number of edges incoming to this vertex, whereas the *out-degree* defines the number of edges outgoing from this vertex. A graph is *balanced*, if for each  $v \in \mathcal{V}$  the in-degree equals the out-degree. A *path* of length  $k$  in a directed graph is a sequence  $v_0, \dots, v_k$  of  $k+1$  distinct vertices such that for every  $i \in \{0, \dots, k-1\}$ ,  $(v_i, v_{i+1}) \in \mathcal{E}$  and a *weak path* is such that  $(v_i, v_{i+1}) \in \mathcal{E}$  or  $(v_{i+1}, v_i) \in \mathcal{E}$ . A directed graph is *strongly connected* if any two vertices can be connected by a path and it is said to be *weakly connected* if any two vertices can be connected by a weak path. An undirected graph can only be connected or disconnected, since there is no distinction between paths and weak paths.

Concerning the underlying network topology of the group of agents, we state

*Assumption 1:* The group of agents with dynamics (1) forms a balanced and static communication network which is (strongly) connected at all times.

This implies that all the agents who are in communication at the beginning, remain connected till the end, i.e. time-varying network topologies are not considered in this article.

### C. Consensus

Consider a set of  $n$  agents forming a communication network, represented by a graph  $\mathcal{G}$  with vertices  $\mathcal{V}(\mathcal{G})$  and edges  $\mathcal{E}(\mathcal{G})$ . Let  $\xi_i(t)$ ,  $i = 1, 2, \dots, n$  be the information state of the  $i$ -th agent at time  $t$ . Consensus means the agreement of all the agents on a common information state by mutual decision and consent [17], i.e.  $|\xi_i - \xi_j| \rightarrow 0$  as  $t \rightarrow \infty$ .

For a kinematic system the continuous time consensus is

$$\dot{\xi}_i(t) = - \sum_{j \in \mathcal{N}_i} g_{ij}(t)(\xi_i - \xi_j), \quad (2)$$

where  $i, j \in \mathcal{V}(\mathcal{G})$ ,  $g_{ii}(t) = 1$  and  $g_{ij}(t) = 1$ ,  $\forall j \neq i$  if information flows from agent  $j$  to agent  $i$  at time  $t$  and 0 otherwise.  $\mathcal{N}_i$  denotes the set of neighbors agent  $i$  can communicate with at time  $t$ .

### D. Potential forces

As significant problems have been encountered using artificial potential fields (as the gradients of potential functions) for the derivation of formation control laws [11], we will now emphasize a way to avoid these problems by employing potential functions introduced in [5].

To this end, consider

$$\mathbf{f}(\mathbf{z}) = -\mathbf{z}[f_a(\|\mathbf{z}\|) - f_r(\|\mathbf{z}\|)], \quad (3)$$

where  $f_a: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  represents the magnitude of an attraction term and has a long range, whereas  $f_r: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  represents the magnitude of a repulsion term and has a short range, while  $\|\mathbf{z}\| = \sqrt{\mathbf{z}^T \mathbf{z}}$  represents the Euclidean norm. Additionally, let some constants  $l_a, l_r$  represent the ranges of the attraction and the repulsion term, respectively, and choose  $l_a \geq l_r$ . This implies that on long distances, the attraction force prevails and on short distances the repulsive force dominates. Then there exists a unique constant distance  $\delta$  where the two forces exactly balance  $f_a(\delta) = f_r(\delta)$ , while  $f_a(\|\mathbf{z}\|) > f_r(\|\mathbf{z}\|)$  holds  $\forall \|\mathbf{z}\| > \delta$ , and  $f_a(\|\mathbf{z}\|) < f_r(\|\mathbf{z}\|)$  holds for  $\forall \|\mathbf{z}\| < \delta$ .

Note, that the functions  $\mathbf{f}(\cdot)$  are odd, i.e.  $\mathbf{f}(-\mathbf{z}) = -\mathbf{f}(\mathbf{z})$ , and that  $-\mathbf{z}f_a(\|\mathbf{z}\|)$  represents the actual attraction, whereas  $\mathbf{z}f_r(\|\mathbf{z}\|)$  represents the actual repulsion and they both act along the line connecting the two agents. The vector  $\mathbf{z}$  determines this line and guarantees that the interaction forces act along the line of  $\mathbf{z}$  as can be seen in the definition of the function  $\mathbf{f}(\cdot)$  above.

## III. RENDEZVOUS

In this section we will derive a controller for the system (1), facilitating rendezvous of the group at a point. The control laws are based on consensus algorithms and will be proven to work.

### A. Controller design

Consider  $n$  agents scattered at different locations in a plane with random orientations. The agents exchange information comprising the position of the neighboring agents. This is the case where all agents consent upon a common point which is

the average of their initial positions and gather at that point. In order to make the headings converge, take

$$\omega_i = - \sum_{j \in \mathcal{N}_i} (\theta_i - \phi_{ij}),$$

as a consensus based control law for the angular velocity input of each vehicle. Similar to [3]

$$\phi_{ij} = \arctan\left(\frac{y_j - y_i}{x_j - x_i}\right) \quad (4)$$

lets the heading of agent  $i$  point into the direction of the agents in its neighborhood  $\mathcal{N}_i$ . In order to make the agents rendezvous at a point, consider the attractive potential function

$$g_a(\|\mathbf{d}_i\|) = \frac{1}{2}\|\mathbf{d}_i\|^2 = \frac{1}{2}\mathbf{d}_i^\top \mathbf{d}_i, \quad (5)$$

with

$$\mathbf{d}_i = \begin{pmatrix} x_i - x_j \\ y_i - y_j \end{pmatrix}.$$

Then, consensus could be achieved using

$$u_i = - \sum_{j \in \mathcal{N}_i} \nabla g_a^\top \mathbf{t}_i = - \sum_{j \in \mathcal{N}_i} [(x_i - x_j) \cos \theta_i + (y_i - y_j) \sin \theta_i], \quad (6)$$

$$\omega_i = - \sum_{j \in \mathcal{N}_i} (\theta_i - \phi_{ij}),$$

as a controller for each agent with dynamics given by (1).  $\mathbf{t}_i$  denotes the unit heading vector of agent  $i$  as depicted for one vehicle in Fig. 1. The operator  $\nabla(\cdot)$  computes the gradient of a function, which we consider to be a column vector, explaining the  $(\cdot)^\top$  operation.

Note that while adjusting their headings w.r.t. the neighboring agents, all agents approach each other such that the velocity  $u_i \rightarrow 0$  as the agents tend to rendezvous. Thus, the equilibrium for the aggregate system, consisting of  $n$  agents with dynamics as in (1), with the controller (6) is  $\mathbf{d}_{i,e} = \mathbf{0}$  and  $\theta_{i,e} = \phi_{ij}$ .

The results for the rendezvous case are shown in Fig. 2. Four agents consent on one point in the plane and converge towards it, while their headings consent and converge to  $\theta_i = 0, \forall i$ . We will analyse in the next section how this behavior emerges from the given equations. Note that the initial conditions for all the results in this paper are  $\mathbf{x}_0 = [-10, 12, 10, -1]^\top$ ,  $\mathbf{y}_0 = [12, -12, 8, -15]^\top$  and  $\theta_0 = [\frac{\pi}{4}, -\frac{\pi}{2}, \frac{2\pi}{3}, \pi]^\top$  and are marked by “o”.

### B. Stability analysis

We will investigate the stability of the aggregate system consisting of  $n$  copies of (1) with the rendezvous controller (6) resulting in the dynamics

$$\begin{aligned} \dot{\mathbf{d}}_i &= - \sum_{j \in \mathcal{N}_i} \mathbf{H}_i \mathbf{d}_i, \\ \dot{\theta}_i &= - \sum_{j \in \mathcal{N}_i} \left[ \theta_i - \arctan\left(\frac{y_j - y_i}{x_j - x_i}\right) \right], \end{aligned} \quad (7)$$

where

$$\mathbf{H}_i = \begin{pmatrix} \cos^2 \theta_i + \cos^2 \theta_j & \cos \theta_i \sin \theta_i + \cos \theta_j \sin \theta_j \\ \cos \theta_i \sin \theta_i + \cos \theta_j \sin \theta_j & \sin^2 \theta_i + \sin^2 \theta_j \end{pmatrix}.$$

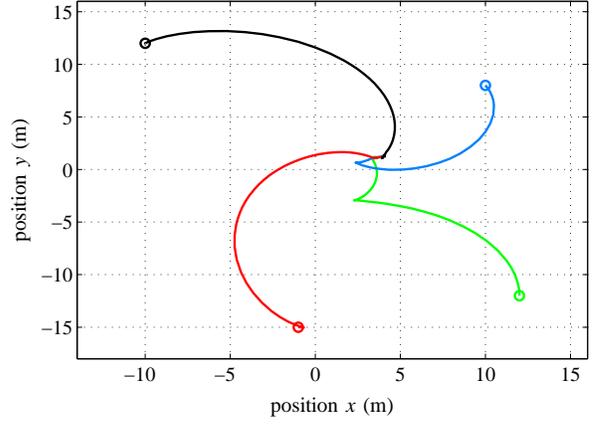


Fig. 2. Phase plot of simple rendezvous of four agents with controls (6)

Additionally, we used that  $\mathbf{d}_j = -\mathbf{d}_i$  holds for balanced graphs.

Consider the continuously differentiable function

$$V = \sum_{i=1}^n V_i = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \frac{1}{2} \mathbf{d}_i^\top \mathbf{d}_i.$$

defined on a compact set  $\mathcal{B}$  w.r.t. (7). Then, the derivative of  $V_i$  w.r.t. the time  $t$  is

$$\dot{V}_i = \sum_{j \in \mathcal{N}_i} \mathbf{d}_i^\top \dot{\mathbf{d}}_i = - \sum_{j \in \mathcal{N}_i} \mathbf{d}_i^\top \mathbf{H}_i \mathbf{d}_i,$$

As  $\mathbf{H}_i$  is symmetric and positive semidefinite, we have that

$$\dot{V} = - \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \mathbf{d}_i^\top \mathbf{H}_i \mathbf{d}_i \leq 0$$

in the set  $\mathcal{B}$ . Employing the LaSalle-Krasovskii invariance principle [9], we constrain (7) to the set  $\mathcal{S} = \bigcup_{i=1}^n \mathcal{S}_i = \bigcup_{i=1}^n \{\mathbf{d}_i, \theta_i \in \mathcal{B} | \dot{V}_i \equiv 0\}$ . Clearly, this means  $\mathbf{d}_i \equiv \mathbf{0}$  or, due to the semidefiniteness of  $\mathbf{H}_i$ ,  $\theta_i = \theta_j, \forall j \in \mathcal{N}_i$ . Regarding the dynamics (7) w.r.t.  $\theta_i = \theta_j$  one can see that, due the definition of  $\phi_{ij}$  in (4), the dynamics can not stay identically in  $\mathcal{S}$ , while with the former condition,  $\mathbf{d}_i \equiv \mathbf{0}$ , we obtain

$$\begin{aligned} \dot{\mathbf{d}}_i &= \mathbf{0}, \\ \dot{\theta}_i &= - \sum_{j \in \mathcal{N}_i} \left[ \theta_i - \arctan\left(\frac{y_j - y_i}{x_j - x_i}\right) \right]. \end{aligned}$$

According to [3], we further define  $\arctan\left(\frac{0}{0}\right) = 0$  resulting in

$$\dot{\theta}_i = - |\mathcal{N}_i| \theta_i,$$

where  $|\mathcal{N}_i|$  gives the cardinality of  $\mathcal{N}_i$ . This subsystem clearly is asymptotically stable, so we conclude that the largest invariant set contained in  $\mathcal{S}$  is the trivial set  $\mathcal{M} = \bigcup_{i=1}^n \mathcal{M}_i = \bigcup_{i=1}^n \{\mathbf{d}_i, \theta_i \in \mathcal{B} | \mathbf{d}_i \equiv \mathbf{0} \wedge \theta_i \equiv 0\}$ , implying asymptotic stability of the aggregate motion and consensus is reached.

#### IV. FORMATION CONTROL

In this section we will extend the results from rendezvous to formation control for the group of unicycles. Further we will distinguish between formations set up using global, i.e. inertial information, and formations that are set up using local information only.

Generally, formations are set up using the potential forces defined in Section II-D. The controller in (6) is modified in order to accommodate an attractive-repulsive function instead of a pure attractive function. The attractive function used is the same as in (5) while the repulsive function is, according to [2], simply put as

$$g_r(\|\mathbf{d}_i\|) = \frac{1}{2} \exp(c - \|\mathbf{d}_i\|^2). \quad (8)$$

This function exactly equals the attractive potential when  $\|\mathbf{d}_i\|^2 = c$ . Hence,  $c$  can be appropriately adjusted to get the desired clearance between the agents.

##### A. Formation control using global information

Having inertial information available the modified controller is given by

$$\begin{aligned} u_i &= - \sum_{j \in \mathcal{N}_i} \nabla(g_a + g_r)^\top \mathbf{t}_i \\ &= - \sum_{j \in \mathcal{N}_i} [(x_i - x_j) \cos \theta_i + (y_i - y_j) \sin \theta_i] k_d(\mathbf{d}_i), \quad (9) \\ \omega_i &= - \sum_{j \in \mathcal{N}_i} (\theta_i - k_d(\mathbf{d}_i) \phi_{ij}), \end{aligned}$$

where

$$k_d(\mathbf{d}_i) = 1 - \exp(c - \|\mathbf{d}_i\|^2). \quad (10)$$

This controller makes the headings  $\theta_i \rightarrow 0$  as the difference in linear positions  $\|\mathbf{d}_i\|^2 \rightarrow c$ . At the equilibrium point, all the agents in communication consent on the distance  $c$  from each other and align parallel to each other with  $\theta_i = 0$ . As a result, this controller could be used to generate formations.

Next, we want to show how the group of agents may travel in formation, based on the control law in (9). Moreover, this time we advance the controller in (9) in that the communication is minimized by using a ring-like communication topology, i.e. each agent receives information from one neighbor and transmits information to one other agent in the group. They then make a polygonal formation pattern and converge to a linear distance of  $c$  from their respective neighbors. In order for the agents to move ahead keeping the polygonal pattern intact, the controller is set to

$$\begin{aligned} u_i &= -[(x_i - x_j) \cos \theta_i + (y_i - y_j) \sin \theta_i] k_d(\mathbf{d}_i) + u_d, \quad (11) \\ \omega_i &= -(\theta_i - k_d(\mathbf{d}_i) \phi_{ij}) + (k_d(\mathbf{d}_i) - 1) \theta_d, \end{aligned}$$

where  $u_d$  and  $\theta_d$  are the desired fixed velocity and angle, respectively, which could be dynamically changing to accommodate trajectory following. Here the term  $u_d$  adds an offset constant velocity to the agents without affecting the stability of the system and  $(k_d(\mathbf{d}_i) - 1) \theta_d$  makes the headings of all the agents converge to the desired angle  $\theta_d$  in the inertial frame as the agents approach the point of rendezvous. The sequence of all the events happening is: i) All agents move

toward each other facing the other agents, ii) they converge around a point of rendezvous maintaining a desired clearance  $c$  between them, iii) after making the rendezvous they align parallel to each other facing in the direction of the desired orientation  $\theta_d$  and finally, iv) the agents move ahead with a fixed absolute velocity maintaining the formation with the desired orientation.

Results for the controller in (11) are shown in Fig. 3. Starting from an initial distribution four agents set up a formation pattern and track a desired reference, where we set  $\theta_d = \frac{\pi}{4}$ ,  $u_d = 0.5$  m/s and  $c = 8$  m.

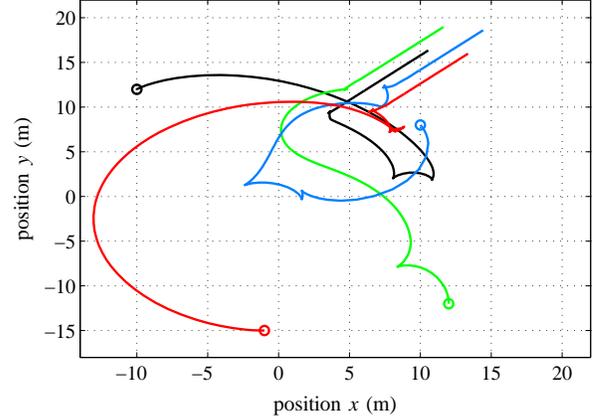


Fig. 3. Formation keeping for a group of four agents with controls (11)

##### B. Formation control using local information

In the previous subsections, we developed a control strategy for rendezvous and formation keeping of multiagent systems in an inertial frame. The formation keeping was shown as an extension of rendezvous. Next, we move on to show the rendezvous without any inertial information available, resulting in an invariant (w.r.t. the chosen inertial frame) control law. System description is based on [8] and is depicted in Fig. 4.

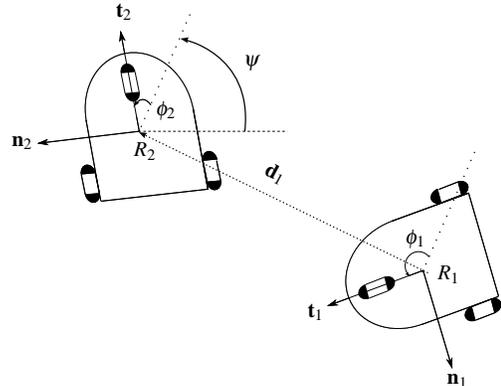


Fig. 4. Body fixed frames for two vehicles

Describing the dynamics w.r.t. a frame  $[\mathbf{t}_i, \mathbf{n}_i]$  fixed at the center of mass of each unicycle, a so called Frenet-Serret-

frame, we have that

$$\begin{aligned}\dot{\mathbf{r}}_i &= u_i \mathbf{t}_i, \\ \dot{\mathbf{t}}_i &= \omega_i \mathbf{n}_i, \\ \dot{\mathbf{n}}_i &= -\omega_i \mathbf{t}_i.\end{aligned}\quad (12)$$

Each agent senses the position of its nearest neighbor agent without a direct communication. According to the model, the necessary information is the angle of the vector connecting the two agents and the absolute distance between the agents. For this system the aim is to make a rendezvous in the plane. Also, as in the previous system, the potential functions from (5) and (8) are utilized to avoid collisions and stabilization of the agents with a desired clearance between them, achieving a circular formation pattern. The controller for this system is given by

$$\begin{aligned}u_i &= \frac{\mathbf{d}_i^\top}{\|\mathbf{d}_i\|} \mathbf{t}_i = -k_d(\mathbf{d}_i) \sin \phi_i, \\ \omega_i &= \frac{\mathbf{d}_i^\top}{\|\mathbf{d}_i\|} \mathbf{n}_i = \cos \phi_i,\end{aligned}\quad (13)$$

where a similar definition as in (9) is used for  $u_i$ . With this controller, the agents move in the direction of their respective neighbors and make a rendezvous in a circular formation pattern around the point of rendezvous, which is shown in Fig. 5, where  $c = 1$  m.

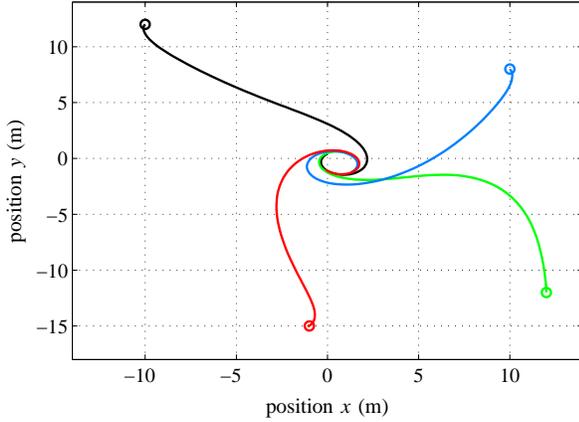


Fig. 5. Formation building using the invariant control law in (13)

We omit further stability analysis in this section, since similar arguments as in Section III-B could be used with the equilibrium transformed to  $\|\mathbf{d}_i\| = c$ .

## V. OBSTACLE AVOIDANCE

In this Section we want to extend our controller with the ability to avoid obstacles while the agents are driving in a formation. To this end, we switch to a subroutine enabling the avoidance when an obstacle is detected.

We define an obstacle to be a rigid body which comes in the path of motion of the agent and which cannot be communicated to. With this definition, we clearly differentiate between avoiding an obstacle in the path and avoiding collisions with other agents. Moreover, obstacles are detected

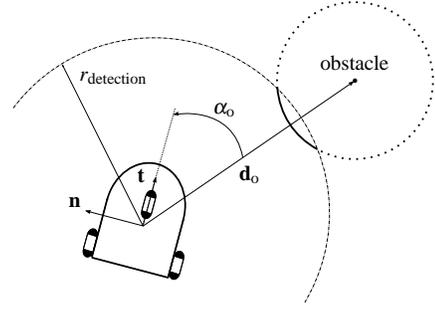


Fig. 6. Relevant parameters for detecting an obstacle

by sensing, whereas the agents additionally have information exchange to help avoiding collisions.

Concerning obstacle avoidance for mobile robots, we solely consider obstacles that have a convex surface, that each agent has a sufficient sensing radius to be able to detect the obstacle early enough to maneuver itself around it, and that each agent has a  $360^\circ$  field of view, i.e. even when moving backwards the agents will be able to detect obstacles. Hence, each agent must have an angular velocity  $\omega_i$  sufficiently higher than its linear velocity  $u_i$  in order to trace a trajectory with minimal radius.

The obstacle comes into *detection* if it is inside the sensing range of the agent and the agent is moving in the direction of the obstacle. This situation is shown in Fig. 6. The controller performs its regular functions, i.e. making rendezvous and formation keeping as long as there is no obstacle detected. In the event of a detection, a new algorithm takes over and no formation keeping or rendezvous takes place. The new controller in event of a *detection* is,

$$\begin{aligned}u_i &= k_u \sin \alpha_o, \\ \omega_i &= \cos \alpha_o,\end{aligned}\quad (14)$$

where  $k_u$  is a constant velocity that is low enough for the agent to take a trajectory with radius short enough to be able to avoid the obstacle and  $\alpha_o$  is the angle between the heading of each agent  $\theta_i$  and the vector of minimum distance to the obstacle from the agent  $\mathbf{d}_o$ . These definitions

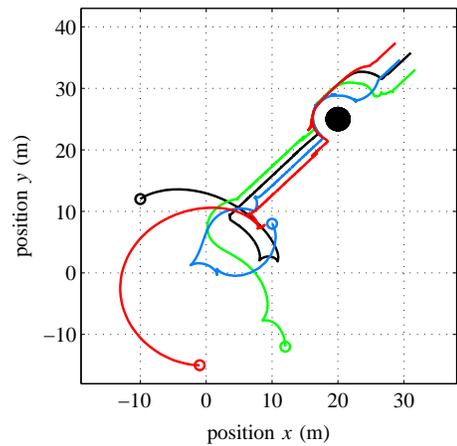


Fig. 7. Obstacle avoidance for the group with controls (14)

are clarified in Fig. 6. Concerning the control law in (14),  $\omega_i = \cos \alpha_o$ , changes the heading of each agent in a direction perpendicular to the distance vector  $\mathbf{d}_o$  and  $\sin \alpha_o$  acts as a factor weighting the influence of the fixed velocity  $k_u$ .

As detection criteria, choose

$$\|\mathbf{d}_o\| \leq r_{\text{detection}} \quad \wedge \quad u_i \cos \alpha_o > 0,$$

to switch to the obstacle avoidance routine of the controller, otherwise it follows the formation keeping routine. The system is shown to work in Fig. 7, where we used the same parameters as in Section IV-A.

## VI. COMPARISON

In order to classify our results, a short comparison to the approaches in [3], [4], [7],[12], [13] will be given in this section. All of these either solve the formation control or rendezvous problem or both.

In [3] only the rendezvous case is considered. The derived controller is somewhat similar to the one in (6). In particular the same definition for the angular velocity is used. But the main difference is that the applied control law is discontinuous, having the advantage of a faster convergence rate compared to time-varying feedback laws [10]. Moreover, the derived controllers could be used to include a connectivity maintenance control, where each of the agents who is in communication at the beginning remains in communication till the end and the control laws also work (proven using nonsmooth analysis) for dynamic network topologies.

In [4] rendezvous and target-point tracking are considered. The derivation of the control laws is based on a chained form (cf. [15]) for the dynamics in (1). In addition to a dynamic network topology, the given control laws also work in networks with communication delays.

The work [7] derives controllers for the rendezvous and the formation control problem. The control laws are also based on potential functions but have the structure of dynamic compensators, as for each of the control variables  $u_i, \omega_i$  a variable, dynamically extending the controller, is introduced. Directed graphs describing the network topology of the group are included in their proofs. Finally, it is shown how circular formation patterns could be generated by using an invariant manifold description of the formations.

In [12] general conditions on the network topology stabilizing a group of unicycles are derived, while the approach excludes dynamic network topologies. Moreover, it is shown that arbitrary formation patterns could be generated using the proposed time-varying control laws. The derivation of the control laws is based on the local Frenet-Serret frame description in (12). Therefore, the definition for the angular velocity in (13) is similar, but a different velocity input  $u_i$  is used. Stability is proven using tools from averaging theory.

Last, in [13] control laws based on cyclic pursuit are derived that enable regular polygons as formation patterns. These formation patterns are stable equilibria for control laws achieving circular pursuit. The description is again based on a local Frenet-Serret frame. The case of rendezvous is not considered.

The main drawback of our approach is the tight constraint we met in our proof concerning the network topology. We see that approaches exist, where the derived control laws will also stabilize the systems if the network topology is dynamically changing. Novel in our approach is the combination of a rendezvous and a formation controller using the same control scheme. This is based on an idea in [16], where it was proposed to work for having simple integrator dynamics. Additionally, we provide a means to avoid obstacles keeping the formation intact.

## VII. CONCLUSION

We presented a rendezvous and formation controller based on consensus algorithms and artificial potential fields, stabilizing the aggregate motion of a group of unicycles. Moreover, we added obstacle avoidance (based on switching to a subroutine) to each team member, allowing to react to changes in the environment while the group is navigating autonomously.

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